VECTORS.

1. **Position Vector. (p.v)**

Given A, B, C are pts
O is fixed point.

$\bar{a} = \overrightarrow{OA}$ $\quad \bar{b} = \overrightarrow{OB}$ $\quad \bar{c} = \overrightarrow{OC}$

are p.v of A, B, C resp.

$\overrightarrow{AB} = \bar{b} - \bar{a}$ $\quad \overrightarrow{AC} = \bar{c} - \bar{a}$

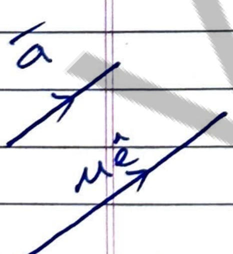
In 3-D

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ are unit vector along x, y, z axis resp.

$$|\bar{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

unit vector along $\bar{a} = \dfrac{\bar{a}}{|\bar{a}|}$



$$\hat{e} = \frac{a_1\hat{i} + a_2\hat{j} + a_3\hat{k}}{\sqrt{a_1^2 + a_2^2 + a_3^2}}.$$

vector of magnitude m is

$= m\hat{e}$

$$= \frac{m}{\sqrt{a_1^2 + a_2^2 + a_3^2}} \times (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$$

$\bar{a}$ & $\bar{b}$ are vectors in 3-D θ is angle between them

$$\cos\theta = \frac{\bar{a}\cdot\bar{b}}{|\bar{a}|\,|\bar{b}|}$$

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$

$\bar{a}\cdot\bar{b} = a_1b_1 + a_2b_2 + a_3b_3$

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{\sqrt{a_1^2+a_2^2+a_3^2}\cdot\sqrt{b_1^2+b_2^2+b_3^2}}$$

If $\bar{a}\cdot\bar{b}$ $\cancel{\cos\theta}$ = +ve angle is acute

$\bar{a}\cdot\bar{b}$ $\cancel{\cos\theta}$ = −ve " is obtuse

$\bar{a}\cdot\bar{b}$ $\cancel{\cos\theta}$ = 0 angle is 90°.

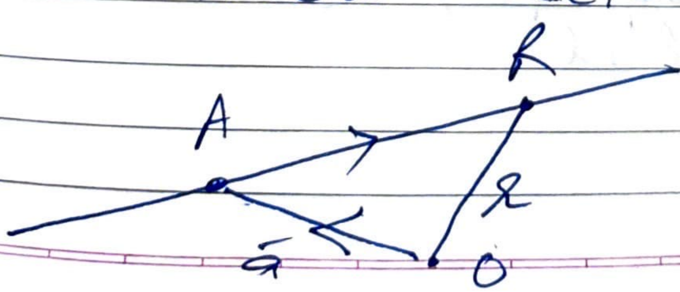
If $\bar{a} = k\bar{b}$ vectors are parallel.

Vector Equation of line

Equation of line $\bar{r} = \bar{a} + \lambda(\bar{d})$

$\bar{a}$ – point on line

$\bar{d}$ – direction vector of line



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for Any line

vector equation:

$$\bar{r} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k} + \lambda(d_1\hat{i} + d_2\hat{j} + d_3\hat{k})$$

Column vector

$$\bar{r} = \begin{pmatrix} a_1 + d_1\lambda \\ a_2 + d_2\lambda \\ a_3 + d_3\lambda \end{pmatrix}$$

parallel lines

If $\vec{r} = a + \lambda d_1$

$\vec{r} = b + \mu d_2$

If $d_1 = k d_2$ lines are parallel

$$\vec{r} = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$

$$\vec{r} = \begin{pmatrix} 3 \\ 4 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -3 \\ 3 \end{pmatrix}$$

If $d_1 \neq k d_2$ lines are not parallel.

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If two lines intersect then they have common point

$$\vec{r} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} d_1 \\ d_2 \\ d_3 \end{pmatrix} = \begin{pmatrix} a_1 + \lambda d_1 \\ a_2 + \lambda d_2 \\ a_3 + \lambda d_3 \end{pmatrix}$$

$$\vec{r} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} + \mu \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} = \begin{pmatrix} b_1 + \mu D_1 \\ b_2 + \mu D_2 \\ b_3 + \mu D_3 \end{pmatrix}$$

$a_1 + \lambda d_1 = b_1 + \mu D_1$
$a_2 + \lambda d_2 = b_2 + \mu D_2$

Solve to get μ & λ

Subs. in $a_3 + \lambda d_3 = b_3 + \mu D_3$
if L.H.S = R.H.S
lines intersect if not then lines do not

Note: To prove skew lines

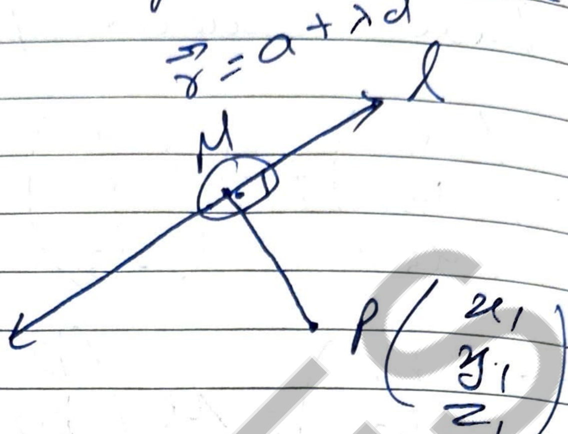
Step 1 Show $d_1 \neq k d_2$ ✓

Step 2 Show lines do not intersect

lines are not parallel
& lines do not intersect
∴ Skew

$\perp^r$ distance of point from line

$$\vec{r} = \begin{pmatrix}a_1\\a_2\\a_3\end{pmatrix} + \lambda\begin{pmatrix}d_1\\d_2\\d_3\end{pmatrix}$$



$$\vec{r} = \begin{pmatrix}a_1+\lambda d_1\\a_2+\lambda d_2\\a_3+\lambda d_3\end{pmatrix}$$

Draw PM $\perp$ line l M lies on line l

$$\therefore\ m = \begin{pmatrix}a_1+\lambda d_1\\a_2+\lambda d_2\\a_3+\lambda d_3\end{pmatrix} \qquad p = \begin{pmatrix}x_1\\y_1\\z_1\end{pmatrix}$$

$$\vec{PM} = \cancel{a_1+\lambda d_1 - x_1}\ \cancel{a_2+\lambda d} \quad \begin{pmatrix}a_1-x_1+\lambda d_1\\a_2-y_1+\lambda d_2\\a_3-z_1+\lambda d_3\end{pmatrix}$$

(d.v)

$\vec{PM}$. line l = 0

$$\therefore \begin{pmatrix}a_1-x_1+\lambda d_1\\a_2-y_1+\lambda d_2\\a_3-z_1+\lambda d_3\end{pmatrix} \cdot \begin{pmatrix}d_1\\d_2\\d_3\end{pmatrix} = 0$$

Solve to get λ
Subs λ in PM.
find $|PM|$

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To find foot of $\perp^r$
Subs. λ in M.

To find reflection of P^t under line l.

Subs. λ in M & find $\underline{M}$ co-ord

$\therefore$ M is midpoint of PP' $\begin{pmatrix} m_1 \\ m_2 \\ m_3 \end{pmatrix}$

$\frac{x_1+x}{2} = m_1$ $\quad$ $\frac{y_1+y}{2} = m_2$ $\quad$ $\frac{z_1+z}{2} = m_3$